



Generation of geometrical phases and persistent spin currents in 1-dimensional rings by Lorentz-violating terms

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ABSTRACT

We have demonstrated that Lorentz-violating terms stemming from the fermion sector of the SME are able to generate geometrical phases on the wave function of electrons confined in 1-dimensional rings, as well as persistent spin currents, in the total absence of electromagnetic fields. We have explicitly evaluated the eigenenergies and eigenspinors of the electrons modified by the Lorentz-violating terms, using them to calculate the dynamic and the Aharonov–Anandan phases in the sequel. The total phase presents a pattern very similar to the Aharonov–Casher phase accumulated by electrons in rings under the action of the Rashba interaction. Finally, the persistent spin current were carried out and used to impose upper bounds on the Lorentz-violating parameters.

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1. Introduction

The standard model extension (SME) [1] was proposed as an extension of the minimal standard model including terms of Lorentz symmetry violation in all interaction sectors. Its gauge sector has been much investigated in several respects [2–6], including photon–fermion interactions [7], nonminimal couplings with higher order derivatives [8], and higher dimension operators [9]. The initial investigations on the fermion sector of the SME were associated with the breaking of the CPT symmetry [10], its consistency, stability, hermiticity, quantization respects [11], its nonrelativistic regime, Foldy–Wouthuysen transformation [12], and some modified Dirac equations [13]. The fermion sector of the SME is described by the following Lagrangian,

$$\mathcal{L}_{\text{Total}} = \frac{1}{2} i \bar{\psi} \Gamma^\nu \overleftrightarrow{\partial}_\nu \psi - \bar{\psi} M \psi, \quad (1)$$

where the field ψ is a Dirac spinor, and

$$\Gamma^\nu = \gamma^\nu + c^{\mu\nu} \gamma_\mu + d^{\mu\nu} \gamma_5 \gamma_\mu + \frac{1}{2} g^{\lambda\mu\nu} \sigma_{\lambda\mu}, \quad (2)$$

$$M = m + a_\mu \gamma^\mu + b_\mu \gamma_5 \gamma^\mu - \frac{1}{2} H_{\mu\nu} \sigma^{\mu\nu}, \quad (3)$$

with the terms $c^{\mu\nu}$, $d^{\mu\nu}$, $g^{\lambda\mu\nu}$, a_μ , b_μ and $H_{\mu\nu}$ standing for Lorentz-violating (LV) tensors, which have the following mass di-

mension: $[a_\mu] = [b_\mu] = [H_{\mu\nu}] = 1$, $[c^{\mu\nu}] = [d^{\mu\nu}] = [g^{\lambda\mu\nu}] = 0$. While a_μ and b_μ break the CPT symmetry, the terms $c^{\mu\nu}$, $d^{\mu\nu}$ and $H_{\mu\nu}$ are CPT-even. The modified Dirac equation is

$$[i\Gamma^\nu \partial_\nu - M] \psi = 0, \quad (4)$$

whose nonrelativistic regime was analyzed in Ref. [12]. The physical effects induced by these parameters are used to state upper bounds on their magnitude [14,15].

Condensed matter systems constitute a proper environment in which Lorentz-violating theories may find application, once are usually endowed with rotation invariance breakdown or privileged directions. Two dimensional electron systems have demonstrated to be a rich field where spin–orbit interaction plays a very relevant role in connection with the spintronics, where geometrical phases and persistent currents are remarkable observables. Geometrical phases in quantum mechanics have been much investigated since Berry's seminal demonstration about the phase accumulated in a cyclic adiabatic evolution [16], and the Aharonov–Anandan discovery about the geometrical phase developed in a cyclic nonadiabatic evolution [17]. Observable effects of geometrical phases in condensed matter systems have been reported since the 80s, as the conductance oscillations due to Aharonov–Bohm effect in mesoscopic systems [18]. In the early 90s, it was shown that the motion of electrons in mesoscopic rings in the presence of magnetic field implies the generation of Berry phase [19]. In 1992, Mathur & Stone [20] showed that conductance oscillations in semiconductors are a signature of the Aharonov–Casher effect [21]. Balatsky & Altshuler [22] investigated the motion of electrons in

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one-dimensional rings under the action of the electric field responsible by the Aharonov–Casher effect. Aronov & Lyanda-Geller [23] investigated the motion of electron in conducting rings, showing that the spin–orbit interaction gives origin to a Berry phase in an adiabatic cyclic evolution. Qian & Su [24] argued that electrons in a nonadiabatic cyclic evolution in mesoscopic rings acquire an Aharonov–Casher phase composed of a dynamic and an Aharonov–Anandan phase, which in the adiabatic limit recovers the Berry phase obtained by Aronov & Lyanda-Geller. Implications involving the geometrical phases associated with persistent currents [25] and transport properties in mesoscopic rings [26] were addressed in several works [19,22,27–37].

The Rashba interaction on electrons confined in a 1-dimensional ring can be addressed as stated in Ref. [26]. The starting Hamiltonian is $H = \frac{1}{2m} [\mathbf{p} - \mu(\boldsymbol{\sigma} \times \mathbf{E})]^2$, where the electric field is $\mathbf{E} = E(\cos \chi \hat{r} - \sin \chi \hat{z})$, the term $\mathbf{p} \cdot (\boldsymbol{\sigma} \times \mathbf{E})$ represents the Rashba interaction, and $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ represents the Pauli matrices. The corresponding eigenenergies are

$$E = \frac{1}{2mr_0^2} \left[n - \frac{\Phi_{AC}}{2\pi} \right]^2, \quad (5)$$

where r_0 is the radius of the ring, and $\Phi_{AC} = -\pi(1 - \lambda_{\pm})$ is the Aharonov–Casher phase developed by the eigenspinor,

$$\Psi^{(\pm)} = e^{in\varphi} \begin{bmatrix} \cos(\beta_{\pm}/2) \\ \pm \sin(\beta_{\pm}/2) e^{i\varphi} \end{bmatrix}, \quad (6)$$

after a complete cycle. Here, $\lambda_{\pm} = \pm \sqrt{\omega_1^2 + (\omega_3 + 1)^2}$, $\beta_{-} = \pi - \beta_{+}$, $\tan \beta_{\pm} = \omega_1/(\omega_3 + 1)$, $\omega_1 = (\mu E r/\hbar c) \sin \chi$, $\omega_3 = (\mu E r/\hbar c) \cos \chi$. From the eigenspinors (6), one evaluates a dynamic phase, $\Phi_{dyn}^{(\pm)}$, and the Aharonov–Anandan geometrical phase, $\Phi_{AA}^{(\pm)}$, whose sum yields the total AC phase that appears in Eq. (5).

Investigations about the generation of topological phases by Lorentz-violating terms have begun in the context of a CPT-odd nonminimal coupling between photons and fermions [38]. New studies involving Aharonov–Bohm, Aharonov–Casher and topological phases in scenarios endowed with Lorentz symmetry breaking were developed in Refs. [39,40]. The interesting similarity between Lorentz-violating terms of the SME fermion sector, regarded in the nonrelativistic limit, and some spin–orbit interactions of the condensed matter systems, as the Rashba term, $H_{Rashba} = \alpha_R(\sigma_y p_x - \sigma_x p_y)$, and the Dresselhaus interaction, $H_D = \beta(\sigma_x p_x - \sigma_y p_y)$, were discussed in recent works [41–44]. It was verified that the nonrelativistic limit of the mass term, $H_{\mu\nu} \sigma^{\mu\nu}$, leads to the Rashba spin–orbit interaction, that appears as a consequence of an inversion asymmetry potential in a semiconductor interface, in the presence of an electric field [31]. In this case, the tensor component H_{0i} corresponds to the Rashba coupling constant (α_R): $\alpha_R = H_{0i}/m$, so that H_{0i} plays the role of the electric field, E . The Rashba term has appeared also in the context of the Dirac equation modified by a CPT-even nonminimal coupling [42] and a CPT-odd nonminimal coupling [43]. Recently, it was discussed that the term $d^{\mu\nu}$ can recover the Dresselhaus interaction [41].

The purpose of this work is to show that the tensor background, $d^{\mu\nu}$, provides nonrelativistic contributions to the Hamiltonian of electrons confined in a 1-dimensional ring, which alter the corresponding eigenenergies and eigenspinors in a compatible way with the generation of geometrical phases analogue to the ones produced by the Rashba interaction in condensed matter systems. It occurs in the entire absence of electric or magnetic fields. These phases are also associated with induced persistent spin currents, which constitute a feasible route to constrain the Lorentz-violating parameters in mesoscopic systems at a level much better than previously supposed [41].

2. Induction of geometrical phases and persistent spin currents by Lorentz-violating terms

We now investigate the effects played by some terms stemming from the fermion sector of the SME on the wave function of electrons confined in 1-dimensional rings, pointing out that we are using natural units, $\hbar = 1$, $c = 1$. Particularly, we are interested in the Dirac equation, in the absence of electromagnetic field,

$$(i\Gamma^\nu \partial_\nu - m) \Psi = 0, \quad (7)$$

where

$$\Gamma^\nu = \gamma^\nu + d^{\mu\nu} \gamma^5 \gamma_\mu, \quad (8)$$

with $d^{\mu\nu}$ being the CPT-even, symmetric and traceless tensor belonging to fermion sector of the SME. The nonrelativistic Hamiltonian associated with Eq. (7), obtained under the condition $|\mathbf{p}|^2 \ll m^2$, is

$$H = \frac{\mathbf{p}^2}{2m} + md_{j0}\sigma^j + d_{jk}p^j\sigma^k + d_{00}p^j\sigma^j - \frac{3}{2}d_{0j}\frac{p_j p^l \sigma^l}{m} + \frac{\mathbf{p}^2}{2m^2}d_{mj}p^j\sigma^m + d_{jk}\frac{p_j p_k}{2m^2}p^l \sigma^l. \quad (9)$$

The effective nonrelativistic Hamiltonian, composed of the more meaningful terms, is

$$H = \frac{\mathbf{p}^2}{2m} - md^j\sigma^j + d_{jk}p^j\sigma^k + d_{00}p^j\sigma^j, \quad (10)$$

where $d^j = -d_{j0}$. The term $d^j\sigma^j = \mathbf{d} \cdot \boldsymbol{\sigma}$ acts providing a Zeeman-like effect or a magnetic moment contribution in the case the vector \mathbf{d} could play the role of a magnetic field. This term will not be considered anymore, since we focus attention on the terms $d_{jk}p^j\sigma^k$ and $d_{00}p^j\sigma^j$ that provide analogue structures to the Rashba and Dresselhaus interactions. The tensor $d^{\mu\nu}$ is CPT-even, and its elements can be classified under the action of the discrete operations: P (parity), C (charge conjugation), T (time reversal). All them are C -odd, and PT -odd. The elements d^{00} , d^{ij} are T -even and P -odd, while the coefficients d^{0i} are T -odd and P -even. The T -even character of d^{00} , d^{ij} will allow to obtain persistent spin current but no charge current [25,29,31] in the next sections.

2.1. Phases and spin currents generated by the coefficients d^{ij}

We begin our investigation doing $d^{00} = d^{0i} = 0$, and keeping our attention in the term $d_{jk}p^j\sigma^k$, so that the Hamiltonian (10) becomes

$$H = \frac{\mathbf{p}^2}{2m} + d^{ij}\sigma^i p^j, \quad (11)$$

presenting a general structure analogue to the Rashba and Dresselhaus interactions. The term $d^{ij}\sigma^i p^j$ can be explicitly written as

$$d^{ij}\sigma^i p^j = d^{11}\sigma_x p_x + d^{22}\sigma_y p_y + d^{12}(\sigma_x p_y + \sigma_y p_x). \quad (12)$$

Here, we have used $d^{12} = d^{21}$, and taken $p_z = 0$, once the electron moves on the plane. We have also set $d^{32} = d^{31} = d^{33} = 0$, so that d^{ij} becomes a 2×2 matrix. Choosing $d^{22} = -d^{11}$, the tensor $d^{\mu\nu}$ becomes traceless, as required. Considering that the electron moves on a ring of fixed radius, $r = r_0$, we can write the momentum in polar coordinates,

$$p_x = \frac{\sin \varphi}{r_0} \left[i \frac{\partial}{\partial \varphi} \right], \quad p_y = -\frac{\cos \varphi}{r_0} \left[i \frac{\partial}{\partial \varphi} \right], \quad (13)$$

so that

$$d^{ij}\sigma^i p^j = \alpha_{11} (\sigma_x \sin \varphi + \sigma_y \cos \varphi) \left[i \frac{\partial}{\partial \varphi} \right] + \alpha_{12} (-\sigma_x \cos \varphi + \sigma_y \sin \varphi) \left[i \frac{\partial}{\partial \varphi} \right], \quad (14)$$

where $\alpha_{11} = (1/r_0) d^{11}$, $\alpha_{12} = (1/r_0) d^{12}$.

The Hamiltonian (11), in polar coordinates, is

$$H = \Omega \left[i \frac{\partial}{\partial \varphi} \right]^2 + \alpha_{11} (\sigma_x \sin \varphi + \sigma_y \cos \varphi) \left[i \frac{\partial}{\partial \varphi} \right] + \alpha_{12} (-\sigma_x \cos \varphi + \sigma_y \sin \varphi) \left[i \frac{\partial}{\partial \varphi} \right], \quad (15)$$

with $\Omega = 1/2mr_0^2$. We can observe that the term $d^{ij}\sigma^i p^j$ yields a contribution equal to the Dresselhaus interaction,

$$H_{\text{Dresselhaus}} = \alpha_D (\sigma_x \sin \varphi + \sigma_y \cos \varphi) \left[i \frac{\partial}{\partial \varphi} \right], \quad (16)$$

and one analogue, but not equal, to the Rashba interaction,

$$H_{\text{Rashba}} = \alpha_R (\sigma_y \sin \varphi + \sigma_x \cos \varphi) \left[i \frac{\partial}{\partial \varphi} \right]. \quad (17)$$

When the Hamiltonian (11) is written in polar coordinates, as Eq. (15), it lets to be hermitian, as pointed out in Refs. [28,31]. In order to find its hermitian form, we follow the procedure of Ref. [31], achieving

$$H = \Omega \left[i \frac{\partial}{\partial \varphi} \right]^2 + [\alpha_{11} (\tilde{\sigma}_\rho) - \alpha_{12} (\tilde{\sigma}_\varphi)] \left[i \frac{\partial}{\partial \varphi} \right] + \frac{i}{2} [\alpha_{11} (\tilde{\sigma}_\varphi) + \alpha_{12} (\tilde{\sigma}_\rho)], \quad (18)$$

where

$$\tilde{\sigma}_\rho = \sigma_x \sin \varphi + \sigma_y \cos \varphi, \quad (19)$$

$$\tilde{\sigma}_\varphi = \sigma_x \cos \varphi - \sigma_y \sin \varphi, \quad (20)$$

with $\tilde{\sigma}_\varphi = \partial_\varphi \tilde{\sigma}_\rho$, $\tilde{\sigma}_\rho = -\partial_\varphi \tilde{\sigma}_\varphi$, and

$$\tilde{\sigma}_\rho = \begin{bmatrix} 0 & -ie^{-i\varphi} \\ ie^{i\varphi} & 0 \end{bmatrix}, \quad \tilde{\sigma}_\varphi = \begin{bmatrix} 0 & e^{i\varphi} \\ e^{-i\varphi} & 0 \end{bmatrix}. \quad (21)$$

Except for a constant term, $(\alpha_{11}^2 + \alpha_{12}^2) / (2\hbar\Omega)^2$, the Hamiltonian (18) can be compactly read as

$$H = \Omega \left[i \frac{\partial}{\partial \varphi} + \frac{1}{2\Omega} [\alpha_{11} (\tilde{\sigma}_\rho) - \alpha_{12} (\tilde{\sigma}_\varphi)] \right]^2, \quad (22)$$

the form to be considered henceforth.

Now, we should evaluate the eigenenergies of electrons governed by the nonrelativistic Hamiltonian (22). We solve this problem for H' , given as

$$H' = \left[i \frac{\partial}{\partial \varphi} + [\beta_{11} (\tilde{\sigma}_\rho) - \beta_{12} (\tilde{\sigma}_\varphi)] \right], \quad (23)$$

that is, $H'\Psi = \Lambda\Psi$, so that the eigenenergies are $E = \Omega\Lambda^2$. Here, $\beta_{12} = \alpha_{12}/2\Omega$, $\beta_{11} = \alpha_{11}/2\Omega$. This operator has the matrix form,

$$H' = \begin{bmatrix} i \frac{\partial}{\partial \varphi} & -\beta e^{i(\varphi+\delta)} \\ -\beta e^{-i(\varphi+\delta)} & i \frac{\partial}{\partial \varphi} \end{bmatrix}, \quad (24)$$

where

$$\beta = \sqrt{\beta_{12}^2 + \beta_{11}^2}, \quad \tan \delta = \frac{\beta_{11}}{\beta_{12}}, \quad (25)$$

or

$$\beta = mr_0 \sqrt{d_{12}^2 + d_{11}^2}. \quad (26)$$

The eigenspinors have the general form,

$$\Psi_{n,\lambda}(\varphi) = e^{i\lambda n\varphi} \tilde{\Psi}(\varphi) = e^{i\lambda n\varphi} \begin{bmatrix} a \\ be^{-i\varphi} \end{bmatrix}, \quad (27)$$

with $n \in \mathbb{Z}$, $\lambda = \pm 1$. Solving the equation, $H'\Psi(\varphi) = \Lambda\Psi(\varphi)$, we find eigenenergies

$$E_n = \Omega \left\{ \lambda n - \frac{1}{2} \left[1 + (-1)^\mu \sqrt{1 + 4\beta^2} \right] \right\}^2, \quad (28)$$

associated with spin down ($\mu = 1$) or spin up ($\mu = 2$) states. This energy expression can be read as

$$E_n = \Omega \left[\lambda n - (-1)^\mu \frac{\Phi_{\text{Total}}^{(\mu)}}{2\pi} \right]^2, \quad (29)$$

where

$$\Phi_{\text{Total}}^{(\mu)} = (-1)^\mu \pi \left[1 + (-1)^\mu \sqrt{1 + 4\beta^2} \right] \quad (30)$$

is the total phase induced on the electron eigenspinors,

$$\Psi_{n,\lambda}^{(\mu)} = e^{i\lambda n\varphi} \tilde{\Psi}^{(\mu)}(\varphi), \quad (31)$$

by the Lorentz-violating term, analogue to the Aharonov–Casher phase induced by the Rashba term [24–27,31,35]. This total phase is composed of a dynamic part and a geometrical contribution, as it will be seen as follows.

The normalized eigenspinors, $\tilde{\Psi}^{(\mu)}(\varphi)$, corresponding to the eigenenergies (29),

$$\tilde{\Psi}^{(1)}(\varphi) = \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) e^{-i(\delta+\varphi)} \end{bmatrix}, \quad (32)$$

$$\tilde{\Psi}^{(2)}(\varphi) = \begin{bmatrix} \sin(\theta/2) \\ -\cos(\theta/2) e^{-i(\delta+\varphi)} \end{bmatrix}, \quad (33)$$

represent the spin down and spin up, respectively. Here, we also have

$$\tan(\theta/2) = -\frac{1}{\beta} \left[\frac{1}{2} - \frac{1}{2} \sqrt{1 + 4\beta^2} \right], \quad (34)$$

$$\sin^2(\theta/2) = \left(1 - \sqrt{4\beta^2 + 1} \right)^2 \Xi, \quad (35)$$

$$\sin \theta = -4\beta \left(1 - \sqrt{4\beta^2 + 1} \right) \Xi, \quad (36)$$

$$\cos \theta = 1/\sqrt{1 + 4\beta^2}, \quad (37)$$

$$\text{where } \Xi = \left[\left(\sqrt{4\beta^2 + 1} - 1 \right)^2 + 4\beta^2 \right]^{-1}.$$

Now, we evaluate the phases developed by the spinors $\Psi_{n,\lambda}^{(\mu)}$: the dynamic phase (dyn), the Aharonov–Anandan (AA) geometrical phase, and the total one. We first determine the phases associated with the eigenspinor $\Psi_{n,\lambda}^{(1)}$. The AA geometrical phase [17,25] is given by

$$\Phi_{AA}^{(1)} = i \int_0^{2\pi} \left[\tilde{\Psi}^{(1)}(\varphi) \right]^\dagger \frac{d\tilde{\Psi}^{(1)}(\varphi)}{d\varphi} d\varphi, \quad (38)$$

which leads to

$$\Phi_{AA}^{(1)} = 2\pi \sin^2(\theta/2). \quad (39)$$

On the other hand, the dynamic phase [16,25,26] is given by

$$\Phi_{dyn}^{(1)} = - \int_0^{2\pi} [\tilde{\Psi}^{(1)}(\varphi)]^\dagger H_{eff} \tilde{\Psi}^{(1)}(\varphi) d\varphi, \quad (40)$$

involving

$$H_{eff} = -[\beta_{11}(\tilde{\sigma}_\rho) - \beta_{12}(\tilde{\sigma}_\varphi)], \quad (41)$$

that takes on the matrix form

$$H_{eff} = \begin{bmatrix} 0 & \beta e^{i(\delta+\varphi)} \\ \beta e^{-i(\delta+\varphi)} & 0 \end{bmatrix}. \quad (42)$$

The evaluation implies

$$\Phi_{dyn}^{(1)} = -2\pi \beta \sin \theta. \quad (43)$$

The total phase, $\Phi_{Total}^{(1)} = \Phi_{AA}^{(1)} + \Phi_{dyn}^{(1)}$, acquired by the spinor, $\Psi_{n,\lambda}^{(1)}$, is:

$$\Phi_{Total}^{(1)} = -\pi \left(1 - \sqrt{1 + 4\beta^2}\right). \quad (44)$$

Repeating all evaluations for the spinor $\Psi_{n,\lambda}^{(2)}$, in accordance with the definitions,

$$\Phi_{AA}^{(2)} = i \int_0^{2\pi} [\tilde{\Psi}^{(2)}(\varphi)]^\dagger \frac{d\tilde{\Psi}^{(2)}(\varphi)}{d\varphi} d\varphi, \quad (45)$$

$$\Phi_{dyn}^{(2)} = - \int_0^{2\pi} [\tilde{\Psi}^{(2)}(\varphi)]^\dagger H_{eff} \tilde{\Psi}^{(2)}(\varphi) d\varphi, \quad (46)$$

the generated phases are

$$\Phi_{AA}^{(2)} = 2\pi \cos^2(\theta/2), \quad (47)$$

$$\Phi_{dyn}^{(2)} = 2\pi \beta \sin \theta. \quad (48)$$

The total phase, $\Phi_{Total}^{(2)} = \Phi_{AA}^{(2)} + \Phi_{dyn}^{(2)}$, is

$$\Phi_{Total}^{(2)} = \pi \left(1 + \sqrt{4\beta^2 + 1}\right). \quad (49)$$

Now, we note that it is possible to write the results (44) and (49) as

$$\Phi_{Total}^{(\mu)} = (-1)^\mu \pi \left[1 + (-1)^\mu \sqrt{1 + 4\beta^2}\right], \quad (50)$$

with β given by Eq. (26). Note that it coincides with Eq. (30), justifying the expression (29) for the eigenenergies. Hence, we have shown that the Lorentz-violating term $d^{ij}\sigma^i p^j$ generates, in the entire absence of electromagnetic field, geometrical and total phases analogue to the ones provided by the Rashba coupling in condensed matter systems, as previously discussed in Refs. [25–27,31,35].

In the case the time reversal (T) is a symmetry of the system, there exists no persistent charge current and only persistent spin current can arise [25,29,31]. More details about persistent currents can be found in Refs. [33,45]. As the parameters d^{00} , d^{ij} are T -even and the Hamiltonian (11) is T -symmetric, a nonnull spin current may be induced [31], being defined as

$$\mathcal{J}_z = \Psi_{n,\lambda}^{(\mu)\dagger} \{\mathbf{v}_\varphi, \mathbf{s}_z\} \Psi_{n,\lambda}^{(\mu)}, \quad (51)$$

where $\Psi_{n,\lambda}^{(\mu)}$ is the spinor (31) and $\mathbf{v}_\varphi = ir_0[H, \varphi]$ is the azimuthal velocity along the ring,

$$\mathbf{v}_\varphi = -\frac{i}{mr_0} \partial_\varphi - (d_{11}\tilde{\sigma}_\rho - d_{12}\tilde{\sigma}_\varphi). \quad (52)$$

The measurable current is a kind of average on the degenerate states of the system [33] divided by the dimension of the system, that is,

$$I_z = \frac{1}{2\pi r_0} \langle \mathcal{J}_z \rangle, \quad (53)$$

with

$$\begin{aligned} \langle \mathcal{J}_z \rangle = & [\Psi_{1,+}^{(2)}]^\dagger \{\mathbf{v}_\varphi, \mathbf{s}_z\} \Psi_{1,+}^{(2)} + [\Psi_{0,-}^{(1)}]^\dagger \{\mathbf{v}_\varphi, \mathbf{s}_z\} \Psi_{0,-}^{(1)} \\ & + [\Psi_{-1,-}^{(2)}]^\dagger \{\mathbf{v}_\varphi, \mathbf{s}_z\} \Psi_{-1,-}^{(2)} + [\Psi_{0,+}^{(1)}]^\dagger \{\mathbf{v}_\varphi, \mathbf{s}_z\} \Psi_{0,+}^{(1)}, \end{aligned} \quad (54)$$

being the average spin current carried out on the four degenerate states, $\Psi_{0,-}^{(1)}$, $\Psi_{0,+}^{(1)}$, $\Psi_{1,+}^{(2)}$, $\Psi_{-1,-}^{(2)}$, with energy $\frac{1}{4} [1 - \sqrt{1 + 4\beta^2}]^2$.

Performing the evaluation (54) with

$$\{\mathbf{v}_\varphi, \mathbf{s}_z\} = \frac{1}{mr_0} \begin{bmatrix} -i\partial_\varphi & 0 \\ 0 & i\partial_\varphi \end{bmatrix}, \quad (55)$$

we obtain the spin current density:

$$\mathcal{J}_z = \frac{2}{mr_0} (\cos \theta - 1). \quad (56)$$

Considering the expression (37), and expanding $\cos \theta$ for small β^2 , the measurable current is

$$I_z = \frac{1}{\pi mr_0^2} (\cos \theta - 1) \simeq \frac{2\beta^2}{\pi mr_0^2}, \quad (57)$$

which can be properly used to constrain the magnitude of the d_{ij} coefficients. For a consistency issue, it is possible to demonstrate that the charge current, $\Psi_{n,\lambda}^{(\mu)\dagger} \mathbf{v}_\varphi \Psi_{n,\lambda}^{(\mu)}$, evaluated over the degenerated states regarded in Eq. (54), is really null, as expected.

2.2. Phases and spin currents generated by the coefficient d_{00}

It is possible to show that similar effects are induced by the term $d_{00}\sigma^j p^j$ of the Hamiltonian (10), written as

$$d_{00}\sigma^j p^j = d_{00}(\sigma_x p_x + \sigma_y p_y). \quad (58)$$

In this case, we set $d_{ij} = 0$, and $d_{33} = -d_{00}$, in (10), so that we consider the simple Hamiltonian

$$H = \frac{\mathbf{p}^2}{2m} + d_{00}\sigma^i p^i, \quad (59)$$

which for an electron in a constant radius ring, $r = r_0$, in polar coordinates, is

$$H = \frac{1}{2mr_0^2} \left[i \frac{\partial}{\partial \varphi} \right]^2 + \alpha_{00} (\sigma_x \sin \varphi - \sigma_y \cos \varphi) \left[i \frac{\partial}{\partial \varphi} \right], \quad (60)$$

for $\alpha_{00} = (1/r_0)d^{00}$. The hermitian form of this Hamiltonian is

$$H = \Omega \left[i \frac{\partial}{\partial \varphi} \right]^2 + \alpha_{00} \tilde{\sigma}_\rho \left[i \frac{\partial}{\partial \varphi} \right] + \frac{i}{2} \alpha_{00} \tilde{\sigma}_\varphi, \quad (61)$$

where

$$\bar{\sigma}_\rho = \sigma_x \sin \varphi - \sigma_y \cos \varphi, \quad (62)$$

$$\bar{\sigma}_\varphi = \sigma_x \cos \varphi + \sigma_y \sin \varphi. \quad (63)$$

The Hamiltonian (61) can be read as a squared form

$$H = \Omega \left[i \frac{\partial}{\partial \varphi} + \frac{1}{2\hbar\Omega} \alpha_{00} \bar{\sigma}_\rho \right]^2, \quad (64)$$

except by the term $\alpha_{00}^2/(2\Omega)^2$. Observing that $H = \Omega H''^2$, the Hamiltonian (64) is expressed as

$$H'' = \begin{bmatrix} i \frac{\partial}{\partial \varphi} & i\beta_{00} e^{-i\varphi} \\ -i\beta_{00} e^{i\varphi} & i \frac{\partial}{\partial \varphi} \end{bmatrix}, \quad (65)$$

where

$$\beta_{00} = (mr_0) d_{00}. \quad (66)$$

The eigenenergies are

$$E_n = \Omega \left\{ \lambda n + \frac{1}{2} \left[1 - (-1)^\mu \sqrt{1 + 4\beta_{00}^2} \right] \right\}^2, \quad (67)$$

$$E_n = \Omega \left[\lambda n - \frac{\Phi_{Total}^{(\mu)}}{2\pi} \right]^2, \quad (68)$$

with $n \in \mathbb{Z}$, $\lambda = \pm 1$, and

$$\Phi_{Total}^{(\mu)} = -\pi \left[1 - (-1)^\mu \sqrt{1 + 4\beta_{00}^2} \right]. \quad (69)$$

The eigenspinors have the general form

$$\chi_{n,\lambda}^{(\mu)} = e^{i\lambda n \varphi} \tilde{\chi}^{(\mu)}(\varphi), \quad (70)$$

and explicit solution

$$\tilde{\chi}^{(1)}(\varphi) = \begin{bmatrix} \cos(\vartheta/2) \\ i \sin(\vartheta/2) e^{i\varphi} \end{bmatrix}, \quad (71)$$

$$\tilde{\chi}^{(2)}(\varphi) = \begin{bmatrix} \sin(\vartheta/2) \\ -i \cos(\vartheta/2) e^{i\varphi} \end{bmatrix}, \quad (72)$$

with $\mu = 1$ or $\mu = 2$ representing spin down and spin up, respectively, and

$$\tan(\vartheta/2) = \frac{1}{\beta_{00}} \left[\frac{1}{2} + \frac{1}{2} \sqrt{1 + 4\beta_{00}^2} \right], \quad (73)$$

$$\sin^2(\vartheta/2) = \frac{\left(1 + \sqrt{4\beta_{00}^2 + 1} \right)^2}{\left(\sqrt{4\beta_{00}^2 + 1} + 1 \right)^2 + 4\beta_{00}^2}, \quad (74)$$

$$\cos \vartheta = -1/\sqrt{1 + 4\beta_{00}^2}. \quad (75)$$

Following the same steps of the first case, we obtain the AA and dynamic phases for the spinor $\tilde{\Psi}^{(1)}$:

$$\Phi_{AA}^{(1)} = -2\pi \sin^2(\vartheta/2), \quad (76)$$

$$\Phi_{dyn}^{(1)} = -2\pi \beta_{00} \sin \vartheta, \quad (77)$$

and for the spinor $\tilde{\Psi}^{(2)}$:

$$\Phi_{AA}^{(2)} = -2\pi \cos^2(\vartheta/2), \quad (78)$$

$$\Phi_{dyn}^{(2)} = 2\pi \beta_{00} \sin \vartheta, \quad (79)$$

implying the total phases

$$\Phi_{Total}^{(1)} = -\pi \left(1 + \sqrt{1 + 4\beta_{00}^2} \right), \quad (80)$$

$$\Phi_{Total}^{(2)} = -\pi \left(1 - \sqrt{1 + 4\beta_{00}^2} \right), \quad (81)$$

which are in the pattern of expression (69). Thus, we conclude that the coefficient d_{00} also succeeds in generating geometrical and total phases to electrons in 1-dim rings similar to the ones provided by the Rashba or Dresselhaus interactions.

The measurable current stems from Eq. (53), with spinors $\chi_{n,\lambda}^{(\mu)}$ given Eq. (70) and the average density current explicitly written as

$$\langle \mathcal{J}_z \rangle = \left[\chi_{0,+}^{(2)} \right]^\dagger \{ \mathbf{v}_\varphi, \mathbf{s}_z \} \chi_{0,+}^{(2)} + \left[\chi_{0,-}^{(2)} \right]^\dagger \{ \mathbf{v}_\varphi, \mathbf{s}_z \} \chi_{0,-}^{(2)} + \left[\chi_{1,-}^{(1)} \right]^\dagger \{ \mathbf{v}_\varphi, \mathbf{s}_z \} \chi_{1,-}^{(1)} + \left[\chi_{-1,+}^{(1)} \right]^\dagger \{ \mathbf{v}_\varphi, \mathbf{s}_z \} \chi_{-1,+}^{(1)}, \quad (82)$$

while $\chi_{0,-}^{(2)}$, $\chi_{0,+}^{(2)}$, $\chi_{1,-}^{(1)}$, $\chi_{-1,+}^{(1)}$ are the four degenerate states with energy $\frac{1}{4} \left[1 - \sqrt{1 + 4\beta_{00}^2} \right]^2$.

The evaluation (54) leads to the following spin current density:

$$\mathcal{J}_z = \frac{2}{mr_0} (1 + \cos \vartheta), \quad (83)$$

related to the corresponding persistent spin current

$$I_z = \frac{1}{\pi mr_0^2} (1 + \cos \vartheta) \simeq -\frac{2\beta_{00}^2}{\pi mr_0^2}, \quad (84)$$

or $I_z = -2md_{00}^2/\pi$, to be used in the next section to impose upper limits on the magnitude of the LV parameters.

3. Upper bounds on the LV parameters

An interesting issue concerns the use of these results to obtain upper bounds on the magnitude of the Lorentz-violating coefficients, d^{ij} , d^{00} , responsible for the effects here reported. In accordance with Ref. [14], the best upper bounds on the coefficients d^{ij} reach the level of 1 part in 10^{14} – 10^{15} , being achieved by means of TeV inverse Compton radiation from astronomical sources [15]. There are not tight bounds at all for these coefficients in the context of condensed matter systems. In Ref. [41], there were estimated upper bounds of the order $d^{ij} < 10^{-2}$ by straightforward comparison with the Rashba constant magnitude in electronic systems. However, the phenomenology of electrons in 1-dim rings can provide some mechanisms that can “amplify” the Lorentz-violating effects, leading to much better upper bounds. There are at least two main routes to do it: one involving measurements of topological phases, other related to measurements of persistent spin currents.

The first route to set upper limits in the LV parameters is analyzing the geometrical phases induced in the absence of fields. Since 1989 it is experimentally known the possibility of measuring A-Casher phases as small as 10^{-3} rad [46,48]. There are also several works analyzing the conductance oscillations and transport properties in 1-dim rings endowed with induced A-Casher phase [26,27,30,34,35,37,47]. As in the present work the LV terms predict phase and persistent current induction independently of electromagnetic fields, a suitable experiment to constrain such terms should investigate the generation of geometrical phase and correlated effects on 1-dimensional electrons in the absence of spin-orbit couplings and electromagnetic fields.

We should first consider the total phase yielded by the coefficient d^{00} , given by Eq. (69). Taking $\mu = 1$ and considering $\beta_{00}^2 \ll 1$, we have $|\Phi_{Total}^{(\mu)}| \simeq 2\pi\beta_{00}^2 = 2\pi(mr_0)^2 d_{00}^2$, in accordance with Eq. (66). For electrons of effective mass $m = 0.05m_e$ in a typical mesoscopic ring with $r_0 = 1 \mu\text{m}$, it holds $mr_0 = 1.3 \times 10^5$ [47, 48]. If we consider a mesoscopic ring in the absence of fields and spin-orbit couplings, no phases can be generated at all, so that we state $|\Phi_{Total}^{(\mu)}| < 10^{-3}$ rad. This general condition leads to:

$$|d_{00}| < 1.0 \times 10^{-7}, \quad (85)$$

standing for a bound that can be communicated for the components d^{ij} , once the tensor $d^{\mu\nu}$ is traceless. In the configuration of interest, $|d_{00}| = |d_{33}|$. This bound is not so good as the ones of Refs. [14,15], but we should remark that it is now estimated in the context of a mesoscopic condensed matter system, being much better than the best one suggested in Ref. [41], $d^{ij} \sim 10^{-2}$, by a factor 10^5 . The same procedure can be applied to the total phase (50) provided by the coefficient d_{ij} , implying the same result $|d_{ij}| < 10^{-7}$. As the upper bound is proportional to $1/r_0$, increasing the radius leads to tighter limits, obviously without loosing the mesoscopic character of the system.

Another effective way to constrain the LV parameters is appealing to the spin persistent current associated with this model. It is known that currents as small as 0.1 nA can be measured in mesoscopic rings [48]. Noting that persistent charge and spin currents can be distinguished from each other [33,45], working in a 1-dimensional ring endowed with T -symmetry in the absence of fields and spin-orbit couplings, no spin persistent current can be generated at all. This *gedanken* system allows to impose $2md_{00}^2/\pi < 10^{-10}$ A, which yields

$$d_{00} < 1.4 \times 10^{-6},$$

and similarly, $|d_{33}| < 1.4 \times 10^{-6}$. Here, we used $1\text{A} = 3.52 \times 10^2$ eV. This second route yields a level of constraining one order of magnitude below that latter one but seems to be more confident and realizable, once current measurements are much more precise and accessible than the ones of phases. Even in this case, the bound remains at least four order of magnitude better than the ones estimated in Ref. [41] by direct comparison with the Rash/Dresselhaus couplings.

4. Conclusions

In this work, we have shown that LV terms belonging to the fermion sector of the SME can induce geometrical phases to electrons in 1-dim rings similar to the ones yielded by the Rashba and Dresselhaus interactions in condensed matter systems, in total absence of electromagnetic fields. Particularly, we analyzed the nonrelativistic terms stemming from $\bar{\Psi}d^{\mu\nu}\gamma_5\gamma_\mu\Psi$, belonging to the fermion sector. After carrying out the modified eigenenergies, we have used the evaluated eigenspinors for explicitly computing the geometrical and dynamic phases developed by the electrons. The phases achieved present a similar pattern to the ones induced by the Rashba coupling. The key role of the (absent) electric field is now played by the LV background. We also carried out the spin persistent currents induced by these coefficients, using it to impose the upper bound $d_{00} < 1.4 \times 10^{-6}$, the best one obtained in the context of mesoscopic system until the moment.

Lorentz-violating effects here reported can be also induced by other terms of the SME fermion sector. Analyzing the full nonrelativistic limit of the SME fermion sector, see Ref. [11], we focus on the following Hamiltonian terms,

$$\frac{1}{2}\epsilon_{klm}g_{mlj}p^j\sigma^k, \quad \frac{1}{2}\epsilon_{kjm}g_{m00}p^j\sigma^k, \quad \epsilon_{jkl}\frac{H_{l0}}{m}p^j\sigma^k, \quad (86)$$

that possess the Rashba or Dresselhaus-like form. The first two ones yield analogue effects to the ones ascribed to the coefficient d_{ij} . We should still comment that other terms of the nonrelativistic Hamiltonian could induce similar geometrical phases, but with smaller magnitude by the factor m^{-1} . The Rashba-like contribution of the nonminimal model [42] can also find similar role as the aforementioned terms, but in the presence of electromagnetic field.

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